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Energy-Shaping of Port-Controlled Hamiltonian Systems by Interconnection

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Abstract

Passivity–based control (PBC) has shown to be very powerful to design robust controllers for physical systems described by Euler–Lagrange (EL) equations of motion. The application of PBC in regulation problems of mechanical systems yields controllers that have a clear physical interpretation in terms of interconnection of the system with its environment. In particular, the total energy of the closed–loop is the difference between the energy of the system and the energy supplied by the controller. Furthermore, since the EL structure is preserved in closed–loop, PBC is robust *vis á vis* unmodeled dissipative effects. Unfortunately, these nice properties are sometimes lost when PBC is used in other applications, for instance, in electrical and electromechanical systems. In this paper we further contribute to develop a new PBC theory encompassing a broader class of systems, and preserving the aforementioned energy–balancing stabilization mechanism and the structure invariance, continuing upon our work in [14], [9] and [17]. Towards this end we consider port-controlled Hamiltonian systems with dissipation (PCHD), which result from the network modeling of energy-conserving lumped-parameter physical systems with independent storage elements, and strictly contain the class of EL models.

1 Introduction

The term passivity–based control (PBC) was first introduced in [10] to define a controller design methodology which achieves stabilization by rendering *passive* a suitably defined map. This idea has been very successful to control physical systems described by Euler–Lagrange (EL) equations of motion, which as detailed in [11], includes mechanical, electrical and electromechanical applications. PBC has its roots in the groundbreaking work of Takegaki and Arimoto [16] on state–feedback regulation of fully actuated robot manipulators. For such (so–called simple) mechanical systems the controller design proceeds along two basic stages. First, an *energy shaping* stage where we modify the potential energy of the system in such a way that the new potential energy function has a strict local minimum in the desired equilibrium. Second, a *damping injection* stage where we now modify the dissipation function to ensure asymptotic stability.

PBC has been extended, within the class of simple mechanical systems, to consider regulation with output feedback [12], [15], underactuation [1] and the presence of input constraints [5]. PBC ideas were also applied to electrical and electromechanical systems described by EL models, as well as to solve tracking problems –for a complete set of references see [11]. While in regulation problems for mechanical systems it suffices to shape the potential energy, to address the other applications (even in regulation tasks) we had to modify also the kinetic energy. Unfortunately, this modification could not be achieved preserving the Lagrangian structure. That is, in these cases, the closed–loop –although still defining a passive operator– is no longer an EL system, and the storage function of the passive map does not have the interpretation of total energy. Consequently these designs will not, in general, enjoy the nice features mentioned above (see Section 10.3.1 of [11] for a discussion). Another shortcoming of this EL approach is that the "desired" storage function for the closed–loop map is defined in terms of some error quantities whose physical interpretation is far from obvious.

For an interesting alternative approach to the control of Euler-Lagrange systems which addresses some of these problems we refer to [4]; see also [2].

In our paper [14] we have developed a new systematic technique to achieve energy–shaping and damping injection in PBC for set–point regulation of systems described as *port–controlled Hamiltonian systems with dissipation* (PCHD). An important advantage of this method is that the basic step of PBC of choosing

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the "desired" storage function –being now a true energy function– becomes more natural. We also have that, if the damping satisfies some structural conditions (or if it is zero), the total energy is the "energy–balancing function". In the present paper we take a somewhat complementary point of view by stressing the energy-shaping as *resulting* from the interconnection of the PCHD system (the "plant") with a controller system that is *also* a PCHD system; continuing upon our work in [9]. A more detailed exposition will appear in [17].

PCHD models encompass a very large class of physical nonlinear systems, strictly containing the class of EL models. They result from the network modeling of energy–conserving lumped–parameter physical systems with independent storage elements, and have been advocated in a series of recent papers [8], [7], [18] as an alternative to more classical EL (or standard Hamiltonian) models. Besides capturing the energy balance features of physical systems, as in EL models, there are two key advantages of working with PCH models for PBC: firstly, they allow for a clear identification of the structural properties of the system through the damping and the interconnection matrices, in particular, there is a clear–cut distinction between the internal interconnection structure and the interconnection with the environment –in our case, the control action. Secondly, that the structural obstacles for energy shaping and damping injection are better revealed. In this way, the geometric structure of the state–space of Hamiltonian systems can be profitably used for PBC. For instance, the rank deficiency of the internal interconnection matrix reveals the existence of invariants of motion of the system dynamics, also called Casimir functions. The generation of these Casimir functions through the interconnection with a controller system is the key idea in the developments presented in this paper.

2 Port controlled Hamiltonian systems with dissipation

Network modeling of energy-conserving lumped-parameter physical systems [8] with independent storage elements leads to models of the form –called port controlled Hamiltonian (PCH) systems [7], [18]–

$$\Sigma : \begin{cases} \dot{x} &= J(x)\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^\top(x)\frac{\partial H}{\partial x}(x) \end{cases} \tag{2.1}$$

where $x \in X$ are the energy variables, X is the n-dimensional state space manifold (often $\mathbb{R}^n$), the smooth function $H : X \to \mathbb{R}$ represents the total stored energy, which we assume is bounded from below, and $u, y \in \mathcal{R}^m$ are the port power variables. (All vectors defined in the paper are *column* vectors, even the gradient of a scalar function.) u and y are conjugated variables, for instance currents and voltages in electrical circuits or forces and velocities in mechanical systems. The interconnection structure is captured in the $n \times n$ matrix $J(x)$ and the $n \times m$ matrix $g(x)$, both depending smoothly on the state x. Because of the assumption of energy-conservation, the matrix $J(x)$ is skew-symmetric, that is,

$$J(x) = -J^\top(x), \quad \forall\, x \in X \tag{2.2}$$

The geometric structure of Hamiltonian systems has been thoroughly studied in the literature, we refer the interested reader to [3], [6]. The matrix $J(x)$ defines a generalized Poisson bracket on the state manifold (generalized because it need not satisfy the Jacobi-identity [18]). *Energy-dissipation* is included by terminating some of the ports by resistive elements, see e.g. [18]. Indeed, consider instead of $g(x)u$ in (2.1) a term

$$\begin{bmatrix} g(x) & g_R(x) \end{bmatrix} \begin{bmatrix} u \\ u_R \end{bmatrix} = g(x)u + g_R(x)u_R$$

and extend correspondingly $y = g^\top(x)\frac{\partial H}{\partial x}(x)$ to

$$\begin{bmatrix} y \\ y_R \end{bmatrix} = \begin{bmatrix} g^\top(x)\frac{\partial H}{\partial x}(x) \\ g_R^\top(x)\frac{\partial H}{\partial x}(x) \end{bmatrix}$$

Here u_R, y_R denote the power variables at the ports which are terminated by (linear) resistive elements

$$u_R = -Sy_R$$

for some positive semi–definite symmetric matrix S. Substitution in (2.1) leads to models of the form

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\frac{\partial H}{\partial x}(x) \end{aligned} \tag{2.3}$$

where $R(x) := g_R(x)Sg_R^T(x)$ is a positive semi definite symmetric matrix, depending smoothly on x. One obtains the power balance

$$\begin{aligned} \frac{dH}{dt}(x(t)) &= u^T(t)y(t) - \frac{\partial^T H}{\partial x}(x(t))R(x(t))\frac{\partial H}{\partial x}(x(t)) \\ &\leq u^T(t)y(t) \end{aligned} \tag{2.4}$$

showing *passivity* of the system, if H is bounded from below. We call (2.3) a *port-controlled Hamiltonian system with dissipation* (PCHD). Note that in this case two geometric structures play a role: the internal interconnection structure given by $J(x)$ and an additional resistive structure given by $R(x)$, which is determined by the port structure $g_R(x)$ and the linear constitutive relations $u_R = -Sy_R$ of the resistive elements. Many dynamical properties of (2.3) may be inferred from these two geometric structures, in particular the existence of dynamical invariants *independent* of the Hamiltonian H, called *Casimir functions.* For Casimir fuctions we consider the set of p.d.e.'s

$$\frac{\partial^T C}{\partial x}(x) = [J(x) - R(x)] = 0, \quad x \in X \tag{2.5}$$

implying that the time-derivative of C along solutions of the port-controlled Hamiltonian system with dissipation (2.3) is zero (irrespective of the Hamiltonian H) for $u = 0$. Furthermore, this holds for arbitrary input functions $u(\cdot)$ if additionally $\frac{\partial^T C}{\partial x}(x)g(x) = 0$.

A stronger notion of Casimir functions is obtained by considering functions $C : X \to \mathbb{R}$ which are "Casimir functions" for *both* geometric structures defined by $J(x)$ and $R(x)$ respectively, that is

$$\begin{aligned} \frac{\partial^T C}{\partial x}(x)J(x) &= 0 \\ \frac{\partial^T C}{\partial x}(x)R(x) &= 0 \end{aligned} \tag{2.6}$$

3 Energy-shaping by interconnection

Consider a port controlled Hamiltonian system with dissipation (2.3) regarded as a *plant system to be controlled.* It is a classical result (see e.g. [17]) that the standard feedback interconnection of two passive systems again yields a passive system; a result wich can be used for various stability and control purposes. In the same vein we can consider the interconnection of the plant (2.3) with *another* PCHD system

$$C: \quad \begin{aligned} \dot{\xi} &= [J_C(\xi) - R_C(\xi)]\frac{\partial H_C}{\partial \xi}(\xi) + g_C(\xi)u_C \\ y_C &= g_C^T(\xi)\frac{\partial H_C}{\partial \xi}(\xi) \end{aligned} \qquad \xi \in X_C \tag{3.1}$$

regarded as the *controller*, via the standard feedback interconnection $u = -y_C + e, u_C = y + e_C$, with e, e_C external signals inserted in the feedback loop. The closed-loop system takes the form

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} &= (\underbrace{\begin{bmatrix} J(x) & -g(x)g_C^T(\xi) \\ g_C(\xi)g^T(x) & J_C(\xi) \end{bmatrix}}_{J_{cl}(x,\xi)} \\ &\quad - \underbrace{\begin{bmatrix} R(x) & 0 \\ 0 & R_C(\xi) \end{bmatrix}}_{R_{cl}(x,\xi)}) \begin{bmatrix} \frac{\partial H}{\partial x}(x) \\ \frac{\partial H_C}{\partial \xi}(\xi) \end{bmatrix} \\ &\quad + \begin{bmatrix} g(x) & 0 \\ 0 & g_C(\xi) \end{bmatrix} \begin{bmatrix} e \\ e_C \end{bmatrix} \\ \begin{bmatrix} y \\ y_C \end{bmatrix} &= \begin{bmatrix} g(x) & 0 \\ 0 & g_C(\xi) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x}(x) \\ \frac{\partial H_C}{\partial \xi}(\xi) \end{bmatrix} \end{aligned} \tag{3.2}$$

which again is a port-controlled Hamiltonian system with dissipation, with state space given by the product space $X \times X_C$ and total Hamiltonian $H(x) + H_C(\xi)$. Although H_C can be freely assigned, the plant Hamiltonian H is given, and so it is at first instance not clear how we can effectively shape the total energy. The main idea in order to do so is to investigate the *Casimir functions* of the closed-loop system, especially those relating the state variables ξ of the controller system to the state variables x of the plant system. Indeed, by the *Energy-Casimir method*, see e.g. [6], we may always add any function $P(C_1(x,\xi), C_2(x,\xi), \dots, C_k(x,\xi))$ depending on the Casimirs $C_1(x,\xi), C_2(x,\xi), \dots, C_k(x,\xi))$ to the total energy $H(x) + H_C(\xi)$ in order to shape the total energy for our purposes. In particular, we consider Casimir functions of the form

$$\xi_i - G_i(x) \quad , \quad i = 1, \dots, \dim X_C = n_C \tag{3.3}$$

That means that we are looking for solutions of the p.d.e.'s (with e_i denoting the i-th basis vector)

$$\begin{bmatrix} -\frac{\partial^T G_i}{\partial x}(x) & e_i^T \end{bmatrix} \begin{bmatrix} J(x) - R(x) & -g(x)g_C^T(\xi) \\ g_C(\xi)g^T(x) & J_C(\xi) - R_C(\xi) \end{bmatrix} = 0$$

or written out

$$\begin{aligned} \frac{\partial^T G_i}{\partial x}(x)\,[J(x) - R(x)] - g_C^i(\xi)g^T(x) &= 0 \\ \frac{\partial^T G_i}{\partial x}(x)g(x)g_C^T(\xi) + J_C^i(\xi) - R_C^i(\xi) &= 0 \end{aligned} \tag{3.4}$$

with $\frac{\partial^T G_i}{\partial x}$ denoting as before the gradient vector $\left(\frac{\partial G_i}{\partial x_1}, \dots, \frac{\partial G_i}{\partial x_n}\right)$,and g_C^i, J_C^i, R_C^i denoting the i-th row of g_C, J_C, respectively R_C.

Suppose we want to solve (3.4) for $i = 1, \dots, n$, with $n \leq n_C$ (possibly after permutations of $\xi_1, \dots, \xi_{n_C}$). Defining $G := (G_1, \dots, G_n)^T$, we write (3.4) for $i = 1, \dots, n$, more compactly as

$$\begin{aligned} \frac{\partial^T G}{\partial x}(x)\,[J(x) - R(x)] - \tilde{g}_C(\xi)g^T(x) &= 0 \\ \frac{\partial^T G}{\partial x}(x)g(x)g_C^T(\xi) + \tilde{J}(\xi) - \tilde{R}_C(\xi) &= 0 \end{aligned} \tag{3.5}$$

with $\tilde{g}_C$ denoting the submatrix of g_C composed of the first $\tilde{n}$ rows, and $\tilde{J}_C, \tilde{R}_C$ the submatrix of J_C, respectively R_C composed of its first $\tilde{n}$ rows.

Then post-multiplication of the first equation of (3.5) by $\frac{\partial G}{\partial x}(x)$, and using the second equation, yields

$$\frac{\partial^T G}{\partial x}(x)\,[J(x) - R(x)]\,\frac{\partial G}{\partial x} = \bar{J}_C(\xi) + \bar{R}_C(\xi) \tag{3.6}$$

with $\bar{J}_C(\xi), \bar{R}_C(\xi)$ the $\tilde{n} \times \tilde{n}$ left-upper submatrices of J_C, respectively R_C. Collecting on both sides of (3.6) the skew-symmetric and the symmetric parts we conclude [1] that (3.6) is equivalent to

$$\frac{\partial^T G}{\partial x}(x)J(x)\frac{\partial G}{\partial x}(x) = \bar{J}_C(\xi) \tag{3.7}$$

[1] Recall that if $J_1 + R_1 = J_2 + R_2$, & with J_i skew-symmetric and R_i symmetric, $i = 1, 2$, then $J_1 = J_2$, $R_1 = R_2$.

$$-\frac{\partial^T G}{\partial x}(x)R(x)\frac{\partial G}{\partial x}(x) = \bar{R}_C(\xi) \tag{3.8}$$

However, since by assumption $R(x) \geq 0$, $R_C(\xi) \geq 0$ (and thus any principal submatrix of R_C is also positive semi-definite) (3.8) is equivalent to

$$\begin{array}{r} \frac{\partial^T G}{\partial x}(x)R(x)\frac{\partial G}{\partial x}(x) = 0 \\ \bar{R}_C(\xi) = 0 \end{array} \tag{3.9}$$

Furthermore, since $R(x) \geq 0$ the first line of (3.9) is equivalent to

$$R(x)\frac{\partial G}{\partial x}(x) = 0 \tag{3.10}$$

Summarizing we have obtained:

Proposition 1 *The functions $\xi_i - G_i(x)$, $i = 1,\ldots,\bar{n} \leq n_c$ satisfy (3.4) (and thus are Casimirs of the closed-loop generalized Hamiltonian systems (3.2) for $e = 0, e_c = 0$) if and only if $G = (G_1,\ldots,G_{\bar{n}})^T$ satisfies*

$$\begin{array}{l} \frac{\partial^T G}{\partial x}(x)J(x)\frac{\partial G}{\partial x}(x) = \bar{J}_C(\xi) \\ R(x)\frac{\partial G}{\partial x}(x) = 0 \\ \bar{R}_C(\xi) = 0 \\ \frac{\partial^T G}{\partial x}(x)J(x) = \bar{g}_C(\xi)g^T(x) \end{array} \tag{3.11}$$

Proof Only the last equality remains to be shown. This however directly follows from the first line of (3.5) and (3.10) □

In particular, it follows that the functions $\xi_i - G_i(x), i = 1,\ldots\bar{n}$ are Casimirs of (3.2) for $e = 0, e_C = 0$, if and only if they are Casimirs for *both* the internal interconnection structure $J_{cl}(x,\xi)$ *as well as* for the dissipation structure $R_{cl}(x,\xi)$. Hence the closed-loop port controlled Hamiltonian system with dissipation (3.2) for $e = 0, e_C = 0$ reduces to a PCHD system on any multi-level set $\{(x,\xi)\,|\,\xi_i = G_i(x) + c_i, \quad i = 1,\ldots,\bar{n}\}$, by restricting both J_{cl} and R_{cl} to it.

Let us now consider the special case $\bar{n} = n_C$, in which we wish to relate all the controller state variables $\xi_1,\ldots,\xi_{n_C}$ to the plant state variables x via Casimir functions $\xi_1 - G_1(x),\ldots,\xi_{n_C} - G_{n_C}(x)$.
Denoting $G = (G_1,\ldots,G_{n_C})^T$ this means that G should satisfy (see (3.11))

$$\begin{array}{l} \frac{\partial^T G}{\partial x}(x)J(x)\frac{\partial G}{\partial x}(x) = J_C(\xi) \\ R(x)\frac{\partial G}{\partial x}(x) = 0 = R_C(\xi) \\ \frac{\partial^T G}{\partial x}(x)J(x) = g_C(\xi)g^T(x) \end{array} \tag{3.12}$$

In this case the reduced dynamics on any multi-level set

$$L_C = \{(x,\xi)|\xi_i = G_i(x) + c_i, i = 1,\ldots n_C\} \tag{3.13}$$

can be immediately recognized. Indeed, the x-coordinates also serve as coordinates for L_C. The x-dynamics of (3.2) (with $e = 0, e_C = 0$) is given as

$$\dot{x} = [J(x) - R(x)]\frac{\partial H}{\partial x}(x) - g(x)g_C^T(\xi)\frac{\partial H_C}{\partial \xi}(\xi) \tag{3.14}$$

Using the second and the third equality of (3.12) this can be rewritten as

$$\dot{x} = [J(x) - R(x)]\left(\frac{\partial H}{\partial x}(x) + \frac{\partial G}{\partial x}(x)\frac{\partial H_C}{\partial \xi}(\xi)\right) \tag{3.15}$$

substituting now $\xi = G(x) + c$, and using the chain-rule property for differentiation

$$\frac{\partial H_C(G(x)+c)}{\partial x} = \frac{\partial G}{\partial x}(x)\frac{\partial H_C}{\partial \xi}(G(x)+c) \tag{3.16}$$

we conclude that the dynamics on L_C is given as

$$\dot{x} = [J(x) - R(x)]\frac{\partial H_s}{\partial x}(x) \tag{3.17}$$

with

$$H_s(x) := H(x) + H_C(G(x)+c) \tag{3.18}$$

Thus we see that the interconnection of the plant system (2.3) to the controller system has resulted in another PCHD system with the same interconnection and dissipation structure as before, but with *shaped* Hamiltonian H_s given by (3.18). We summarize this in:

Proposition 2: *Consider the feedback interconnected port-controlled Hamiltonian system with dissipation (3.2) for $e = 0, e_C = 0$. Let $G = (G_1,\ldots G_{n_C})$ satisfy (3.12). Then the reduced dynamics on any multi-level set (3.13) is given as the port-controlled Hamiltonian system with dissipation (3.17), where the shaped Hamiltonian H_s is given by (3.18).*

An interpretation of the shaped Hamiltonian H_s in Proposition 4.3.4 in terms of *energy-balancing* is the following. Since $R_C(\xi) = 0$ by (3.12) the controller Hamiltonian H_C satisfies

$$\frac{dH_C}{dt} = u_C^T y_C \tag{3.19}$$

Hence along any multi-level set L_C given by (3.13), invariant for the closed loop generalized Hamiltonian system with dissipation (3.2) for $e = 0, e_C = 0$

$$\frac{dH_s}{dt} = \frac{dH}{dt} + \frac{dH_C}{dt} = \frac{dH}{dt} - u^T y \tag{3.20}$$

since $u = -y_C$ and $u_C = y$. Therefore

$$H_s(x(t)) = H(x(t)) - \int_0^t u^T(\tau)y(\tau)d\tau + \text{constant} \tag{3.21}$$

and the shaped Hamiltonian H_s is the original Hamiltonian H *minus the energy* supplied to the plant system (2.3) by the controller system (3.1) (modulo a constant; depending on the initial states of the plant and controller).

The reduction of the dynamics of the feedback interconnected generalized Hamiltonian system with dissipation (3.2) for $e_C = 0$ but $e \neq 0$ is a bit more complex. The simplest case is as follows.

Proposition 3 *Consider the feedback interconnected port controlled Hamiltonian system with dissipation (3.2) for $e_C = 0$. Let $G = (G_1, \dots, G_{n_C})$ satisfy (3.12), and additionally assume that*

$$J_C(\xi) = 0, \quad g_C(\xi) \text{ is injective} \tag{3.22}$$

Then the reduced dynamics on any multi-level set (3.13) is given as

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H_s}{\partial x}(x) + g(x)e \\ y &= g^T(x)\frac{\partial H_s}{\partial x}(x) \end{aligned} \tag{3.23}$$

with H_s given by (3.18).

Proof By combining the first and the third equalities of (3.12), together with the assumption $J_C(\xi) = 0$, it follows that $g_C(\xi)g^T(x)\frac{\partial G}{\partial x}(x) = 0$. By assumption of injectivity of $g_C(\xi)$ this implies that $g^T(x)\frac{\partial G}{\partial x}(x) = 0$. Hence

$$\begin{bmatrix} -\frac{\partial^T G}{\partial x}(x) & I_{n_C} \end{bmatrix} \begin{bmatrix} g(x) \\ 0 \end{bmatrix} = 0$$

while in view of (3.16) $y = g^T(x)\frac{\partial H}{\partial x}(x) = g^T(x)\frac{\partial H_s}{\partial x}(x)$.
□

Example 4 A mechanical system with damping and actuated by external forces u is described as a PCHD system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = (\begin{bmatrix} 0 & I_k \\ -I_k & -D(q) \end{bmatrix}) \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ B(q) \end{bmatrix} u \\ y &= B^T(q)\frac{\partial H}{\partial p} \end{aligned} \tag{3.24}$$

For a PCH controller system with identity input matrix there exists a solution $G = (G_1(q), \dots, G_m(q))$ to (3.12) if and only if the columns of the input force matrix $B(q)$ satisfy the integrability conditions $\frac{\partial B_{il}}{\partial q_j}(q) = \frac{\partial B_{jl}}{\partial q_i}(q), \quad i,j = 1, \dots k, \quad l = 1, \dots m$. For a Hamiltonian H consisting of a kinetic energy and a potential energy this amounts to a shaping of the potential energy.

There are a couple of possible *extensions* to the above analysis of the feedback interconnection of a PCHD *plant* system to a PCHD *controller* system. Indeed, one may take the controller PCHD system C given by (3.1) to be *modulated* by the state variables x, which means that J_C, R_C and g_C also allowed to depend on x,, in which case e.g. the conditions (3.12) take the form

$$\begin{aligned} \frac{\partial^T G}{\partial x}(x)J(x)\frac{\partial G}{\partial x}(x) &= J_C(\xi, x) \\ R(x)\frac{\partial G}{\partial x}(x) &= 0 = R_C(\xi, x) \\ \frac{\partial^T G}{\partial x}(x)J(x) &= g_C(\xi, x)g^T(x) \end{aligned} \tag{3.25}$$

Especially, allowing g_C to depend on x yields extra flexibility in the design.

Remark Allowing g_C to depend on x may equivalently be formulated as modifying the standard feedback interconnection (see [14]) $u = -y_C, u_C = y$ to a *state modulated* feedback interconnection.

We conclude that under certain conditions the feedback interconnection of a PCHD system (the "plant") with another PCHD system (the "controller") leads to a reduced dynamics given by another PCHD system (3.17)) (possibly with inputs e and outputs y, cf.(3.23)), for the shaped Hamiltonian H_S given by (3.18), with $G(x)$ a solution of (3.12). From a *state feedback* point of view the dynamics (3.17) could have been directly obtained by a state feedback $u = \alpha(x)$ such that

$$g(x)\alpha(x) = [J(x) - R(x)]\frac{\partial H_C}{\partial x}(G(x) + c) \tag{3.26}$$

Indeed , $\alpha(x)$ is given in explicit form as

$$\alpha(x) = -g_C^T(G(x) + c)\frac{\partial H_C}{\partial \xi}(G(x) + c) \tag{3.27}$$

A state feedback $u = \alpha(x)$ satisfying (3.26) is customarily called a *passivity-based control law*, since it is based on the passivity properties of the original plant system (2.3) and transforms (2.3) into *another* passive system with *shaped* storage functions (in this case H_s). Seen from this point of view we have thus shown that the passivity-based state feedback $u = \alpha(x)$ satisfying (3.26) *can be derived* from the interconnection of the PCHD system (2.3) with a PCHD controller system (3.1). This fact has some favorable consequences. Indeed, it implies that the passivity-based control law defined by (3.26) can be equivalently *generated* as the feedback interconnection of the passive system (2.3) with another passive system (3.1). Hence we can directly invoke the classical passivity theorems to derive properties about the controlled system.In particular, the observation that the passivity-based control (3.26) can be derived in this way implies a natural robustness of the controlled system: the plant system (2.3), the controller system (3.1), as well as any other passive system interconnected to (2.3) in a power-continuous fashion, may change in any way as long as they remain passive, and for any perturbation of this kind the controlled system will still remain stable.

The discussion about the actual *implementation* of the passivity-based control $\alpha(x)$ is somewhat complex. In cases of *analog* design of a controller the interconnection of (2.3) with the PCHD controller system seems the logical option. Furthermore, it may be favorable to avoid an explicit state feedback, but instead to use the dynamics output feedback controller (3.1). On the other hand, in some applications (e.g., robotics) the measurement of the passive output y may pose some problems, while the resulting state feedback $u = \alpha(x)$ is easier to implement.

The problem of *directly* constructing the passivity-based (state feedback) control $u = \alpha(x)$ such that H_S has desired properties has been addressed in [14].

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